

Chapter 1

Introduction to Algebra Enrichment

This packet belongs to _____

LESSON **Challenge**
1-6 Mission Operation

You can create your own operation. Take a symbol, such as \blacktriangle , and tell what that symbol means using the standard operations of $+$, $-$, \times , and \div .

Example

If $x \blacktriangle y = x + 2xy$, find $5 \blacktriangle 3$.

Apply the definition of \blacktriangle . First, show how to substitute the values or expressions that replace x and y . Then, carry out the standard operations to simplify completely.

1. $7 \blacktriangle 2$

2. $2 \blacktriangle 7$

3. $(a \blacktriangle b) + (b \blacktriangle a)$

4. $[a \blacktriangle (4b)] + [(2a) \blacktriangle b]$

Let $a * b = 2a + \frac{b}{2}$. Apply the definition of $*$. Simplify the resulting expression.

5. $(9 * 4) + (3 * 8)$

6. $(r * 4t) + (4r * 2t)$

UPC Codes

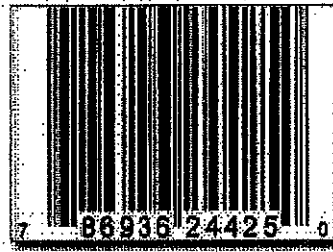
The first patent for barcodes was issued to Bernard Silver and Norman Woodland in 1952. Since then these coding systems have expanded, been modified, and applied to a variety of areas. One of the most common uses is in retail and grocery stores. Although the barcode is not the price of the item, it does allow for the item to be registered with an associated price. When the bar code is scanned, the associated price will be read by the cash register. Other uses are found in monitoring blood supplies, identification on prescription drugs, book checkout at libraries, tracking luggage, and express shipping services.

Another advantage to the barcode system is that when an additional digit is included, known as the *check digit*, many errors that occur during data entry can be detected. These errors occur when passing information over the phone or internet. It is quite easy for people to transpose numbers (45 when it should be 54), replace a single digit with another, omit digits, or double an incorrect digit (799 is entered as 779). Using a check digit within a check equation helps to catch these errors and verify the validity of the number. As students will see in the ISBN barcode, the number is also used for identification purposes.

Begin by taking samples of barcodes that you have collected from items that are sold in stores. This type of barcode is referred to as the Universal Product Code, or UPC.

For the first example, use the given UPC symbol 7-86936-24425-0 from the movie

"The Incredibles."



To verify this number, follow the steps:

1. Every even-positioned digit, counting from the right to left, will be multiplied by 3. All odd-positioned digits will be multiplied by 1.
2. Sum the products.
3. Determine the validity by dividing the sum by 10. If the remainder is 0, the UPC number is valid.

This time, try to figure out the check digit in the following UPC number..

7-96714-78601-y, where y is the check digit.

ISBN numbers

Another barcode system is the International Standard Book Number, or ISBN. This system was developed in the late 1960's and early 1970's. It became apparent that there needed to be a uniform system that would identify books that were published throughout the world. Now every book could have a special identification number. The ISBN is a ten-digit number composed of blocks of numbers that have different meaning. There are four parts to the number, which are separated by hyphens or spaces. The first part of the number identifies the language or country (referred to as the group identifier) and is at most five digits. The second part of the number identifies the publisher and may be at most seven digits. The third part of the number represents the item number or edition for that publisher. It may consist of no more than six digits. The final part is the check digit. Part of the flexibility of this system is the fact that there are many numbers available to be used. Recall that there are a maximum of 10 digits with the 10th being reserved for the check digit. Therefore, the first three parts of the number must have a combined total of nine digits. Leading zeroes are used as place fillers in the event there would not be enough digits in a particular section to ensure there are an appropriate number of digits. The diagram below shows an example of an ISBN number.



The check digit is calculated differently than that of the UPC system. Begin by multiplying the first digit by 10, the second by 9, the third by 8, and continue in this fashion until the ninth digit is multiplied by 2. Next, determine the sum of these products. This is a modulus-11 system, which means that the sum of the products of the first nine digits plus the check digit must be a multiple of 11. One problem that arises in this process is that the check digit might need to be a 10. Because we only have digits 0-9, an X is written in the check-digit place. (The X is reflective of the Roman numeral for 10.)

Note: Beginning January 1, 2007, the current ISBN system will be replaced with the ISBN-13 system (for further information, see the [ISO Web Site](#)). It is a 13-digit number beginning with 978, followed by the current nine digits of the ISBN, and it will have a new check digit. The check digit will be found using a method different from the current one. When all old ISBN's have been used, the next series will begin with 979.

Credit Card numbers

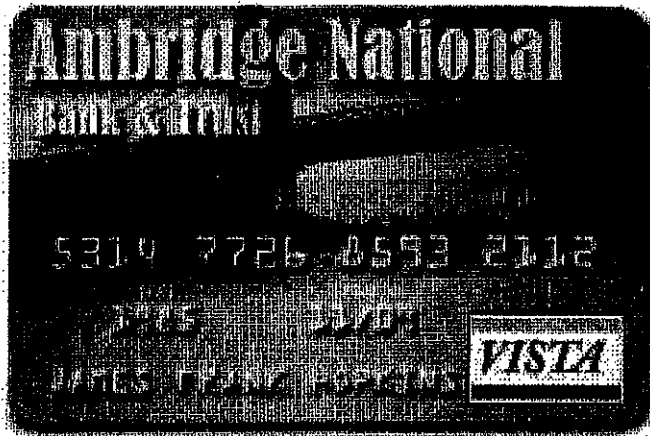
ISSUER	IDENTIFIER	CARD NUMBER LENGTH
Diner's Club/ Carta Blanche	300xxx -- 305xxx 36xxx, 38xxx	14
American Express	34xxx, 37xxx	15
VISA	4xxxx	13, 16
MasterCard	51xxx -- 55xxx	15
Discover	6011xx	16

Credit cards use a system of blocked numbers similar to the ISBN. One obvious difference is that the maximum length for the number is 19 digits, although many numbers range from 13-16 digits.

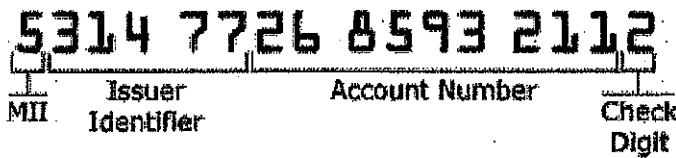
MII DIGIT VALUE	ISSUER CATEGORY
0	ISO/TC 68 and other industry assignments
1	Airlines
2	Airlines and other industry assignments
3	Travel and Entertainment
4	Banking and Financial
5	Banking and Financial
6	Merchandizing and Banking
7	Petroleum
8	Telecommunications and other industry assignments
9	National assignment

The first digit of a credit card number is the Major Industry Identifier (MII) and identifies which group issued the card, as shown below. For instance, a number beginning with a 3 would be representative of the travel and entertainment category. The American Express card falls into this category. Cards issued by gas companies are given the beginning digit 7. The popular Visa and MasterCard fall under the banking and financial category (4, 5). The next block of numbers is the Issue

Identifier. Including the MII digit, the Issue Identifier is six digits long. The account number begins with the seventh digit and ends with the next-to-last digit. The final digit is the check digit.



The process used to determine the check digit is the Luhn algorithm (mod 10), named after IBM scientist Hans Peter Luhn. This algorithm works as follows:



1. Begin by doubling all even-positioned digits when counting from right to left.
2. Determine the sum of the digits from the products (Step 1) and each of the unaffected (odd-positioned) digits in the original number.
3. Verify the account number by determining if the sum from step 2 is a multiple of 10.

To be a valid account number, this sum must be evenly divisible by 10. If the check digit were 1, the result would be congruent to 0 mod 10; but because the check digit is 2, the sum is not divisible by 10.

The Luhn algorithm is able to detect single data entry errors and most transpositions. Students should proceed to the worksheet and determine how this happens.

VIN numbers

The VIN was introduced in its working form in 1983 as the result of a recommendation of the International Organization for Standardization (ISO). This classification is used worldwide to identify motor vehicles, trailers, motorcycles, and mopeds.

Each VIN number is 17 characters long, with three sections. The first three digits are the WMI - World Manufacturer Identifier, the next six digits are the VDS - Vehicle Descriptor Section, and the last eight digits are the VIS - Vehicle Identifier Section.

ISO recommends that the 10th position of the VIN be the year code and the 11th position represent the factory code. All letters other than I, O, and Q are allowed, and the last four digits must be numeric.

The US, however, divides the VIN in four sections (which match up with the three sections of the ISO code):

- The first three digits are the manufacturer code (WMI).
- The next five digits identify the attributes of the vehicle (VDS).
- The next digit is the check digit, an arithmetic check for the other 16 characters. It may be numeric or the letter X.
- The last eight digits are like the VIS above.

Below is a VIN from a Ford Mustang.

1FA CP45E X LF192944
WMI VDS VIS

Purpose of Check Digit in VIN Numbers

A numerical value is assigned to each letter & a weight value is assigned to each position:

A=1	F=6	L=3	S=2	X=7
B=2	G=7	M=4	T=3	Y=8
C=3	H=8	N=5	U=4	Z=9
D=4	J=1	P=7	V=5	
E=5	K=2	R=9	W=6	

1st=8	5th=4	10th=9	14th=5
2nd=7	6th=3	11th=8	15th=4
3rd=6	7th=2	12th=7	16th=3
4th=5	8th=10	13th=6	17th=2

Multiply the numerical value and weight value for each character in each position and add those products. Then divide that sum by 11 and consider the remainder. The remainder is the check digit unless the remainder is 10, in which case the letter X is the check digit.

The VIN number that was provided on an auto loan application is 1B4YEM9P4KP186543.
Is it a valid VIN number?



Check digits

name _____

How does a register at the supermarket accurately scan item numbers? Can mistakes in scanning be detected? The purpose of this activity is to see how modular arithmetic is applied to UPC, ISBN, credit cards and VIN numbers.

- 1) Verify the check digit for each of the two UPC numbers below. Tell whether the number is valid or not. If it is not valid, tell what the check digit should be.
 - a) 0-87684-00974-3
 - b) 0-43197-11682-6

- 2) A problem of the UPC system is that if two adjacent digits that were transposed have a difference of 5, the error is not detected. Explain why this occurs and give an example.

- 3) Verify the check digit for each of the two ISBN numbers below. Tell whether the number is valid or not. If it is not valid, tell what the check digit should be.
 - a) 0-486-20498-7
 - b) 0-87779-621-4

- 4) The ISBN system is better at detecting errors, specifically transposition errors. Question 2 addressed this problem with the UPC system. Explain why, unlike the UPC system, the ISBN system will detect all transposition errors.

- 5) Using the Luhn algorithm, determine the check digit for the following credit card number.
- a) 6011-4387-1005-123_
- 6) In the above process, there are two sums you are finding: one involving the odd-positioned digits and the other involving the even-positioned digits that were doubled. Considering the sum of the odd-positioned digits, what is the most the sum could be affected by if an incorrect digit was entered? Explain why this occurs.
- 7) Now consider the even-positioned digits that were doubled. It is possible to have numbers that result in double digits. Recall how this sum is determined. How is it possible that a single digit error would be detected? [Hint: Make a table which shows possible values for the check digit, the double of the check digit and the resulting sums.]
- 8) The Luhn algorithm is used to detect most digit transpositions. For example, when entering the number 5832403, the data entry error is transposing the second and the third digits: 5382403. There are two digits, when transposed, that will go undetected using the Luhn algorithm. What are they? Explain why this error cannot be detected.

9) Consider the following VIN number recently assigned to a motorcycle. What is the check digit?

a) 1M8GDM9A_KP042788

10) Police found an abandoned car. In order to locate the owner, they took down the VIN number 1PA_GB4AX34134587. The fourth digit was difficult to read. It was either a 6 or a b. Which one is it?