Unit 1 (math 7 and math 7+)

Rational and Irrational Numbers

Standard: 7.EE.4

Use variables to represent quantities in a real world or mathematical problem

Standard:7.NS.3

Solve real-world and mathematical problems involving the four operations with rational numbers

Standard: 7.EE.1

Apply properties of operations as strategies to add, factor and expand linear expressions with rational coefficients

Standard: 7.NS.1 & 7.NS.2

Apply and extend previous understandings of addition and subtraction to add, subtract, multiply & divide rational numbers

8. NS. A 1

All rational numbers have a decimal expansion; converting a decimal expansion that repeats into a rational number

This packet belongs to _____

SWBAT: _____

Standard: 7.NS.1 & 7.NS.2

Apply and extend previous understandings of addition and subtraction to add, subtract, multiply & divide rational numbers.

Use a number line to illustrate:

a) $p - q$

b) $p + (-q)$

c) Is this equation true $p - q = p + (-q)$?

Use a number line to illustrate:

Morgan has $4 and she needs to pay a friend $13. How much will Morgan have after paying her friend? Express your answer as an integer and in real-life terms. Lastly, describe what is happening on a number line.

SWBAT: _____

Standard: 7.EE.4

Use variables to represent quantities in a real world or mathematical problem.

For each of the following problems, describe what is happening on a number line.

1. Two consecutive integers have a sum of 91. What are the two integers?

Steps to solve

1. Label the unknowns
2. Set up the equation
3. Solve the equation
4. Find the value for the unknowns
5. Check
6. Express what is happening on a number line.

2. What three consecutive integers have a sum of 21?

3. What four consecutive integers have a sum of 66?

4. What three consecutive integers have a sum of 33?

5. What two consecutive negative integers have a sum of -47?

6. Two negative consecutive integers have a sum of -61. What is the larger one?

7. Four consecutive integers have a sum of -34. What is the product of the highest and the lowest integers?

Homework--Write an equation to solve each problem. Show all work.

For each of the following problems, describe what is happening on a number line.

1. Two consecutive integers have a sum of 51. What are the two integers?

2. What three consecutive integers have a sum of 171?

3. What four consecutive integers have a sum of 54?

4. What three consecutive integers have a sum of 57?

5. What three consecutive negative integers have a sum of -60?

6. Three negative consecutive integers have a sum of -72. What is the largest one?

7. Four consecutive integers have a sum of -330. What is the product of the highest and the lowest integers?

SWBAT: _____

Standard: 7.EE.4

Use variables to represent quantities in a real world or mathematical problem.

For each of the following problems, describe what is happening on a number line.

1. Find two consecutive even integers such that their sum is 58.

2. Find two consecutive odd integers such that their sum is -48.

3. Find three consecutive even integers such that their sum is 42.

4. Find three consecutive odd integers such that their sum is 75.

5. Find three consecutive odd integers such that their sum is 279.

6. Find three consecutive even integers such that their sum is -132.

7. Find three consecutive even integers such that their sum is -144.

8. Find four consecutive odd integers such that their sum is 64.

9. Find five consecutive even integers such that their sum is 150.

Homework--Write an equation to solve each problem. Show all work.

For each of the following problems, describe what is happening on a number line.

1. Find two consecutive even integers whose sum is 54.

2. Find three consecutive even integers whose sum is 84.

3. Find three consecutive even integers whose sum is −48.

4. Find three consecutive odd integers whose sum is -87.

5. Find three consecutive even integers such that their sum is -180.

6. Find three consecutive even integers such that their sum is -204.

7. Find four consecutive odd integers such that their sum is 120.

8. Find five consecutive odd integers such that their sum is 205.

9. Three consecutive odd integers have a sum of -39. What is their product?

Extension Consecutive Integer Problems

For each of the following problems, describe what is happening on a number line.

1. Find three consecutive odd integers such that the sum of the first and third is the sum of the second and 7.

2. The perimeter of a triangle is 51 centimeters. The lengths of its sides are consecutive odd integers. Find the lengths of all three sides.

3. Find three consecutive integers such that the sum of the first two is three times the third.

4. Find three consecutive odd integers such that twice the sum of the first and third exceeds the second by fifteen.

5. Find three consecutive even integers such that the sum of the second and the third is equal to the first.

6. Three consecutive odd integers are such that the sum of the first and the third is 70. Find the integers.

7. Three consecutive integers are such that three times the smallest is 14 more than the largest. Find the integers.

8. Three consecutive odd integers are such that the sum of the first and second is 31 less than 3 times the third. Find the integers.

9. Three consecutive even integers are such that the sum of the smallest and 3 times the second is 38 more than twice the third. Find the integers.

10. Two consecutive integers are such that 3 times the larger exceeds twice the smaller by 34. Find the integers.

11. John has a board that is 5 feet long. He plans to use it to make 4 shelves whose lengths are to be a series of consecutive even numbers. How long should each shelf be in inches?

SWBAT: _____

Standard: 7.EE.1

Apply properties of operations as strategies to add, factor and expand linear expressions with rational coefficients.

Another property used to simplify expressions is the Distributive Property, which we explore in this lesson. Remember, simplifying does not change the value of the expression. It only changes its appearance.

Consider this: Three groups of musicians are coming to town to perform in a show. Each group consists of 5 women and 4 men. How many total women are there? How many total men?

To find out, you could multiply 5 women and 4 men in each group by 3 groups. There will be a total of 15 women and 12 men. This can be shown algebraically as follows:

3 (5 women + 4 men) = 3 (5 women) + 3 (4 men) = 15 women + 12 men

This is an example of the distributive property of multiplication over addition. The distributive property is used in the multiplication of a number by a sum or difference of two of more numbers.

Try these…

$2(x+3)$ $5(6y+1)$ $7(2c-3)$ $4(5y-3x)$

In the examples shown on the previous page, the number in front of the parentheses is multiplied to each term inside the parentheses. After multiplying, the parentheses can be dropped. It is possible for there to be more than two terms inside the parentheses, as shown in the following examples. You simply distribute to each term.

$6(x + 4y - 1)$ $3(7x - 2y + 5)$ $9(2c - 5y - 3)$

Now you are ready to distribute and then combine like terms. Remember, you can only combine terms with the same base and the same exponent. Try these…

$2(3m - 4) + 7(2m + 5)$ $6(3c - 4) + 2(2c - 9)$

Sometimes the number you are distributing is negative. When that happens, distribute the negative with the number. Try these…

$-2(3x + 3)$ $-5(-6a - 4)$ $-3(2y - 6) - 4(5y - 3x)$

Homework

1. $5(a+9)$
2. $8(2x+1)$
3. $6(4t-3)$
4. $7(3p-2d)$

5. $5(c+3t-5)$
6. $2(6a-d+4)$
7. $-9(4x-7y-5)$

8. $3(4x-6)-6(3x-5)$
9. $-3(5c-2)+4(6c-7)$

10. $-3(5a-6)$
11. $-6(-4a-5)-3(a-7)$
12. $-5(3y-4x)-6(3y-2x)$

SWBAT: _____

Standard: 7.EE.1

Apply properties of operations as strategies to add, factor and expand linear expressions with rational coefficients.

Factoring is the reverse of the distributive property. Imagine you have the rectangle below and you know that its area is equal to 12 ft^2.

What could be the lengths of its sides?

Simple factoring requires you to divide by a common factor.

What if the area of the rectangle was 4a? What could its sides equal then?

Try to find the greatest common factor of each of the following pairs of numbers:

4 and 16 18 and 24 16 and 56

Factor using the slide method

4x + 16 18 - 24a 16c + 56

Here are some more.

$4x + 6$ $3 - 15c$ $8a + 12$ $3a - 12c$

$14a - 7c$ $45x - 9c$ $-4a - 12b$ $-6x - 18d$

You can also factor more than 2 numbers…

$3a - 6c - 9$ $4x - 24b + 12n$ $-6a - 12b + 15 - 18x$

Homework:

Factor THEN distribute when you are done to check

1. $8b + 6$
2. $4 - 12p$
3. $7x + 14y$
4. $6n - 16h$

5. $18n - 4f$
6. $27a - 18c$
7. $-5a - 25j$
8. $-14y - 18g$

9. $-4x + 12v - 8r$
10. $18h - 27t + 36y$
11. $-9a - 12 - 15h - 3b$

Rational Numbers

SWBAT: _____

Standard: 7.NS.2

Apply and extend previous understandings of addition and subtraction to add, subtract, multiply & divide rational numbers

A _____ number is any number that can be written as a fraction $\frac{a}{b}$ where a and b are integers and $b \neq 0$. Let's write some examples of rational numbers.

When you are *simplifying* a fraction, the goal is to make the numerator and the denominator _____. This means that the numerator and the denominator have no common factors other than 1.

You can simplify a rational number by dividing the numerator and the denominator by the same nonzero integer.

Simplify.

1. $\frac{6}{9}$ 2. $\frac{21}{25}$ 3. $\frac{-24}{32}$ 4. $\frac{9c}{12c}$

Decimals that *terminate* or *repeat* are also rational numbers. Write each decimal as a fraction $\frac{a}{b}$ where a and b are integers and $b \neq 0$.

5) .5 6) -2.37 7) .8716 8) .75a

LESSON 3-1

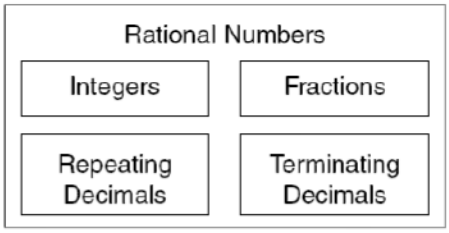
Reteach

Rational Numbers

A **rational number** is a *ratio* of two integers.

$$\text{Rational Number} = \frac{\text{Integer} \leftarrow \text{Numerator}}{\text{Integer} \leftarrow \text{Denominator}}$$

The set of rational numbers contains:
- all integers
- all fractions
- decimals that repeat, such as $0.4\overline{6}$
- decimals that terminate, such as 3.5



To simplify a fraction, divide numerator and denominator by the highest common factor.

$$\frac{5}{15} = \frac{5 \div 5}{15 \div 5} = \frac{1}{3}$$

1) Rational numbers can be categorized in three forms. List the **three forms** of rational numbers & provide examples of each:

- _____ Example: _____
- _____ Example: _____
- _____ Example: _____

2) Irrational numbers are different than rational numbers. What are irrational numbers?

- _____

Give an example of irrational numbers:

- _____

Simplify the following rational numbers (fraction form):

3) $-\frac{21}{39}$	**4)** $\frac{6}{-18}$	**5)** $\frac{-3a}{6a}$	**6)** $\frac{-14}{-32}$

Simplify the following rational numbers by converting the following decimals into fractions. Change mixed numbers into improper fractions so they are in rational form:

7) 0.8	**8)** 2.4	**9)** - 0.35	**10)** -5.06

Simplify the following rational numbers by converting the following fractions to decimals.

11) $\frac{-2}{5}$	**12)** $\frac{2}{-5}$	**13)** $\frac{-11}{-99}$	**14)** $-\frac{7}{8}$

Convert the following fractions to decimals:

1) $-\frac{17}{5}$

2) $\frac{9}{8}$

3) $\frac{-43}{100}$

4) $\frac{7}{-10}$

5) $\frac{2}{5}$

Using long division, express the following fractions as decimals:

$\frac{7}{6}$ $\frac{18}{32}$

$\frac{7}{8}$ $\frac{22}{30}$

$\frac{23}{12}$ $\frac{13}{24}$

$\frac{17}{7}$ $\frac{2}{9}$

Come up with equivalent fractions for the fractions listed:

1) $-\frac{3}{4}$

2) $\frac{-15}{20}$

3) $\frac{5}{-6}$

4) $\frac{-6}{-2}$

5) $\frac{-8h}{-2}$

Homework

Simplify the following fractions:

1) $\frac{6}{8}$

2) $\frac{24}{28}$

3) $\frac{-22}{32}$

4) $\frac{2a}{8a}$

Convert the following decimals to fractions:

5) 0.25

6) – 1.26

7) 0.9753

8) 0.57b

Convert the following fractions to decimals:

9) $\frac{-1}{5}$

10) $\frac{2}{-3}$

11) $\frac{-10}{-80}$

12) $-\frac{1}{8}$

13) $-\frac{16}{5}$

14) $\frac{11}{8}$

Rational vs. Irrational Numbers

SWBAT: _____

Standard: 7.NS.2

Apply and extend previous understandings of addition and subtraction to add, subtract, multiply & divide rational numbers

Let's review:

Remember, all rational numbers fall under two categories: _____ and _____ _____. These can all be written as fractions.

Rational numbers

Rational number (fraction)	Decimal form	Decimal type
$\frac{3}{8}$		
$\frac{2}{7}$		
$\frac{5}{6}$		
$\frac{1}{4}$		

NEW********
Irrational numbers are numbers whose decimal forms are _____ and _____. These ***cannot*** be written as fractions. ☹

Irrational number	Decimal form	Decimal type
π		
	.121121112…	
$\sqrt{2}$		

Note: ***Every rational number can be written as a terminating or a repeating decimal & Every terminating or repeating decimal can be written as a rational number.***

1) Write a statement about how you can determine if a number is rational or not.

_____

_____

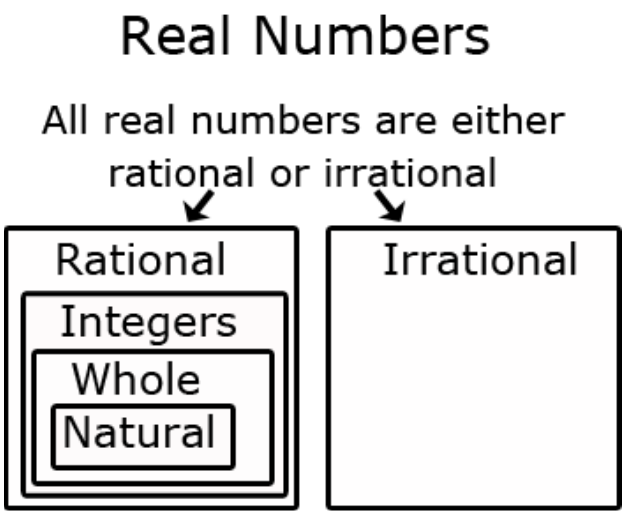
_____

_____

Numbers that do *not terminate* or *repeat* are **irrational numbers**. Use the knowledge you gained from this packet to determine if the numbers below are rational or irrational. Then tell whether each number will terminate or repeat.

2) $\frac{5}{6}$ 3) $\frac{13}{-30}$ 4) $\frac{17}{32}$ 5) $-1\frac{1}{2}$ 6) .121121112

_____ _____ _____ _____ _____



Homework

What are the three forms of rational numbers?

1) _____ Example: _____

2) _____ Example: _____

3) _____ Example: _____

What is an irrational number?

4) _____

Give two examples of irrational numbers?

5) _____

Using long division, express the following fractions as decimals:

6) $\frac{7}{5}$

7) $\frac{14}{32}$

8) $\frac{5}{8}$

9) $\frac{21}{12}$

10) $\frac{15}{7}$

11) $\frac{1}{9}$

Write equivalent fractions for the fractions listed:

12) $-\frac{1}{4}$

13) $\frac{-10}{20}$

14) $\frac{1}{-6}$

15) $\frac{-5}{-2}$

16) $\frac{-10h}{-2}$

1. BOYS AND GIRLS There were 6 girls and 18 boys in Mrs. Johnson's math class. Write the number of girls as a fraction of the number of boys. Then write the fraction as a repeating decimal.	**2. CATS** In a neighborhood of 72 families, 18 families own one or more cats. Write the number of families who own one or more cats as a fraction. Then write the fraction as a decimal.
3. CELLULAR PHONES In Italy, about 74 of every 100 people use cellular telephones. Write the fraction of cellular phone users in Italy. Then write the fraction as a decimal.	**4. FRUITS** Ms. Rockwell surveyed her class and found that 12 out of the 30 students chose peaches as their favorite fruit. Write the number of students who chose peaches as a fraction in simplest form. Then write the fraction as a decimal.
5. TRAVEL Tora took a short trip of 320 miles. He stopped to have lunch after he had driven 120 miles. Write the fraction of the trip he had completed by lunch in simplest form. Then write the fraction as a decimal.	**6. VOTING** In a recent school election, 208 of the 325 freshmen voted in their class election. Write the fraction of freshmen who voted. Then write the fraction as a decimal.

SWBAT: _____

8. NS. A 1

All rational numbers have a decimal expansion; converting a decimal expansion that repeats into a ration number

Changing a repeating decimal into a fraction

You already know how to write a terminating decimal as a fraction. Changing a repeating decimal into a fraction only takes a few more steps. To change a repeating decimal into a fraction, first you have to understand some fractional patterns.

$.\overline{1}$ =	$.\overline{12}$ =	$.0\overline{1}$ =
$.\overline{2}$ =	$.\overline{25}$ =	$.0\overline{2}$ =
$.\overline{3}$ =	$.\overline{67}$ =	$.00\overline{4}$ =
$.\overline{4}$ =	$.\overline{342}$ =	$.00\overline{22}$ =
$.\overline{5}$ =	$.\overline{134}$ =	$.00\overline{81}$ =
$.\overline{6}$ =	$.\overline{6555}$ =	$.0\overline{01}$ =
$.\overline{7}$ =	$.\overline{12134}$ =	$.0\overline{035}$ =
$.\overline{8}$ =	$.\overline{34233}$ =	
$.\overline{9}$ =	$.\overline{98981}$ =	

Change each of these repeating decimals into a fraction. Simplify.

1) $.\overline{2}$ 2) $1.\overline{27}$ 3) $-.1\overline{5}$ 4) $2.3\overline{1}$

5) $-.45\overline{2}$ 6) $8.22\overline{32}$ 7) $-.1123\overline{4567}$

When adding two integers, there are four possible problems you can encounter:

1) Both numbers are positive 3 + 3 or 2 + 56

2) Both are negative -2 + -4 or -5 +-12

3) One is positive and the other is negative -8 + 4 or 3 + -5

4) One of the two numbers is zero 0 + 2 or -7 + 0

Each of these are solved differently. What are some basic rules that apply to these problems?

Examples

$-3+8$ $2+-4$ $-6+-7$ $-1+2+-4+5+-7$

When subtracting integers, change the subtraction sign to an addition sign and change the sign of the number that follows.

Examples

$7-(-3)$ $-4-(-3)$ $0-(+5)$ $-9-(-4)-3-(-7)$

Evaluate $2-(-b)-(-9)$ for $b=-6$

Multiplying and dividing integers

When *multiplying or dividing* several terms, an even number of negative signs will yield a _____ product or quotient, an odd number of negative signs will give you a _____ product or quotient.

Examples

$2 \cdot 4x$ $\quad -3 \cdot -7$ $\quad \frac{6}{-2}$ $\quad \frac{-8h}{-4}$

Rule 1:

Examples

$2 \cdot -4$ $\quad 3 \cdot -7a$ $\quad \frac{-10c}{2}$ $\quad \frac{12}{-4}$

Rule 2:

Remember to always use the order of operations when solving:

Example 1 $\quad 3^2 + 4 \times -2$

Example 2 $\quad 3(-2) \div -2 \times -12$

Example 3 $\quad -3(2) \div 3(-4)$

Complete the problems below.

1. $(1 - 8^2) \times (18 \div 9 - 6)$
2. $3 + (9 - 3^2 + 1 - 1)$
3. $1 + (1 - 9^2) + 4^3 + 4$
4. $(4^2 \div 8) - 5$
5. $6^2 - 1^2 + (4 - 8 - 8)$
6. $3 - (4 - 1) \div 5$
7. $5 \div (-7 \times 1) \div 4$
8. $(2 - 9^2) \div 1^2 - 3$
9. $(8 + 7) \div 1 + 9$
10. $4 \times (1 \div 2^3 + 7)$
11. $6^2 \div -3 \div (-4 + 1) \div 4$
12. $(8^2 \div 2^3) \div 5 + 3$
13. $(3 + -3) - 1$
14. $(3 \div 1 - 6)$
15. $(5^2 \div 5) \div 2$

1) Tracy has $14 and has to pay a friend $8. How much money will Tracy have after paying her friend?

2) Jim's cell phone bill is automatically deducting $42 from his bank account every month. How much will the deductions total for the year?

3) The temperature at 6 a.m. was -15 degrees F, and by noon it had warmed up 8 degrees. What was the temperature at noon?

4) A submarine has an altitude of -5500 feet and it dives down 1500 feet. What is its new altitude in feet?

5) If the temperature is now 30 degrees F, what was it 2 hours ago if it has been decreasing 6 degrees F per hour?

6) Dr. Sarno is working with bacteria samples. She cools one sample of bacteria to a temperature of -51 degrees C and heats another to 76 degrees C. What is the temperature difference between the two samples?

7) Fred had a balance of $112 in his checking account at the beginning of the month. He deposited $384 in the account and then wrote checks for $153, $86, $196, $34, and $79. Then she made a deposit of $123. If at any time during the month the account is overdrawn, a $10 service charge is deducted. At the end of the month, what was Fred's balance?

Homework—Show all work!

1. On Tuesday the mailman delivers 3 checks for $5 each and 2 bills for $2 each. If you had a starting balance of $25, what is the ending balance?

2. You owe $225 on your credit card. You make a $55 payment and then purchase $87 worth clothes at Dillards. What is the integer that represents the balance owed on the credit card?

3. If it is -25F in Rantoul and it is 75F in Honolulu, what is the temperature difference between the two cities?

4. During the football game, Justin caught three passes. One was for a touchdown and went 52 yards. The other was for a first down and was for 17 yards. The other was on a screen pass that did not work so well and ended up a gain of -10 yards. What was the total yardage gained by Justin on the pass plays?

5. James plays in the backfield of the Big Town football team. Last week he ran four plays from the halfback position. He made "gains" measured in yards of 3, 4, 1, and 5. What were his average yards per gain? Round your answer to the nearest tenth of a yard.

6. In golf, the average score a good player should be able to achieve is called "par." Par for a whole course is calculated by adding up the par scores for each hole. Scores in golf are often expressed at some number either greater than or less than par. Ms. Floop is having a pretty good day at the Megalopolis City Golf Club. Her score so far after 15 holes is -3. If par for 15 holes is 63, what is her score?

7. It was a very freaky weather day. The temperature started out at 9°C in the morning and went to -13°C at noon. It stayed at that temperature for six hours and then rose 7°C. How far below the freezing point (0°C) was the temperature at 6 p.m.?

8. A monkey sits on a limb that is 25 ft above the ground. He swings up 10 ft, climbs up 6 ft more, then jumps down 13 ft. How far off the ground is the monkey now?

9. Mary has $267 in her checking account. She writes checks for $33, $65, and $112. What is the balance in her account now?

10. A submarine dove 836 ft. It rose at a rate of 22 ft per minute. What was the depth of the submarine after 12 minutes?

Operations with Variable Terms

Example 1

$3c \cdot 6 + 4(-2c)$

Example 2

$(-12a \div 3) - (-4a)$

Example 3 $2x + 3 - 7x + 6$

Example 4

$3(-4y) \div -6(-2y)$

Example 5

$$\frac{-3a(2) + 3(-4a)}{12a - 6a}$$

Example 6 $(3x + 4x) - 2y$

Complete the problems below.

1. $(5a + 7a) \times 1 - 8a$
2. $(2^3 \times 6x + 4)$
3. $(3z \times 2^2 + 7z \times 2) - 6$
4. $8 - (8a + 3^2 + 6a)$
5. $4x + (3a - 4a + 3x) - 2a$
6. $(8z \times 7) + 8^2 \div 4$
7. $(8a \times 2x \times 4)$
8. $(8a + 9d) - 3a$
9. $(9x - 8) + 9$
10. $7c + (6c \div 2 - 5)$
11. $(2^2 - 7z) + 4z$
12. $3a + (5d - 12a - 3d)$
13. $(6p + 3p \times 5) + 9d$
14. $(6x \div 3) - 2x + 4x$
15. $(9j - 7j) + 4j \div 2$