Unit 5 Math 7+
Exponents and Application

8. EE. A 1: Properties
Know and apply the properties of integer exponents to generate equivalent numerical expressions.

8. EE. A 2: Evaluate Roots (rational/irrational)
Use square root and cube root symbols to represent solutions to equations of the form *x2 = p* and *x3 =p*, where *p* is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.

8. EE. A. 3 and 4: Scientific Notation
Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.

This packet belongs to ____

8. EE. A 1: Properties
Know and apply the properties of integer exponents to generate equivalent numerical expressions.

SWBAT: ____

Exponents: Rules and properties

$$3x^2$$

Rule 1: Anything to the zero power =

Rule 2: For every number a $\neq 0$, $a^m \cdot a^n =$

$9^8 = 3^?$	$8^5 = 2^?$	$2^{18} \cdot 8^6$
$4^{10} = 2^?$	$27^8 = 3^?$	$3^{51} \cdot 9^{22}$
$100^{15} = 10^?$	$64^{211} = 4^?$	$2^{240} \cdot 32^{12}$

Rule 3: For every number a $\neq 0$, $\dfrac{a^m}{a^n} =$

Let's take a look at the some of the pre-test questions:

1. $2^3/5^2$	**2.** $2^2/2^6$	**3.** 6^0
4. $\frac{3^{-2}}{2^4}$	**5.** $(3^2)(3^4)$.	**6.** $(4^3)^2$
7. $\frac{(3^2)^4}{(3^2)(3^3)}$		

SWBAT: ____

Evaluating Expressions with Exponents

Evaluate when $a = 2.5$, $b = 3$, $c = 1.6$ and $d = 2$

1) $a + 2b^3 - c$

2) $b(a - c)^2$

3) $[\,4a + 2(b - c)^2] \div 8$

4) $a^b c^{b+2}$

5) $(a^3 + b - 3c)^0$

6) $a^2 + bc$

7) $(2b)^2[(a + b) \div 11]$

SWBAT: ____

Negative Exponents

2^4	
2^3	
2^2	
2^1	
2^0	
2^{-1}	
2^{-2}	
2^{-3}	
2^{-4}	

Words	Numbers	Algebra
A power with a negative exponent equals __________ by that power with its ______________ exponent.		

1. 3^{-2}
2. -3^{-2}
3. $(-3)^{-2}$
4. 1^{-5}
5. 5^{-1}
6. -5^{-1}
7. $(-2)^{-3}(3)^{-2}$
8. $2^{-3}(2)^{2}$
9. $\frac{1}{5^{-2}}$
10. $\frac{2^{3}}{2^{-3}}$
11. $\frac{3^{-2}}{3^{2}}$

Examples

5^{-3} 2^{-1} x^{-3} $\left[\frac{1}{3}\right]^{-3}$ $(x-2)^{-4}$ $(-.42)^{-2}$

Multiplying examples

a^3a^5 $4^5 4^{-3}$ $3c^2 4c^5$ $-6y(3y^3)$

Dividing examples

$\frac{3^3}{3^2}$ $\frac{x^{-4}}{x^4}$ $\frac{6z^4}{3z^2}$ $\frac{4a^3}{16a^{-2}}$

Raising exponents to a power

$(y^3)^2$ $(3^4)^5$ $(2^2)^{-3}$ $(x^{-3})^{-2}$

$$\frac{(9x^{-2}y^3)^2}{(6xy^2)^3}$$

$$(4x^{-1}y^{-2})^2(2x^4y^5)^3$$

More Practice:

10. $\frac{2^4}{2^{-2}}$

11. $\frac{2x^5}{x^{-2}}$

12. $\frac{2a^{-2}}{4a^2}$

13. $\frac{3x^4}{2x^{-7}}$

14. $(5^2)^5$

15. $(y^3)^{-1}$

16. $(a^{-2})^{-3}$

17. $(c^4)^0$

18. $(x^2)^3$

Please, show your work below.

EXTENTION:

Solving Exponential Equations

Solve each equation by writing each side with the same base.

1) $2^x = 2^5$

2) $5^{3x} = 5^{3+x}$

3) $3^x = 3^{2x+2}$

4) $5^x = 25^3$

5) $4^{x+2} = 64$

6) $9^{2x-5} = 27$

7) $4^{3x+5} = 8^{4x-3}$

8) $5^{x-2} = 1$

9) $9^{2-x} = 81^{6x}$

10) $9^{4x-1} = 27^{5-x}$

11) $2^{x+3} - 1 = 1$

12) $4^{2x} + 1 = 65$

You can use the next page to show work:

Please use this page to show work for problems from the previous page:

13) $3^{x-2} + 5 = 32$

14) $-19 + 5^{2x} = 6$

15) $2^{2+x} \times 16^{x-3} = \frac{4^x}{16^{1+2x}}$

Solve each equation by writing each side with the same base. Use a separate sheet of paper to show your work.

1) $3^x = 9^5$

2) $4^{2x} = 4^{3+x}$

3) $4^x = 16^{2x+2}$

4) $3^{x-2} = 27^3$

5) $5^{x+3} = 125$

6) $4^{2x-5} = 64^{x+2}$

7) $64^{3x+5} = 2^{4x+3}$

8) $3^{x-2} = 1$

9) $6^{2x-5} = 216^{4x}$

10) $3^{x-1} + 4 = 13$

11) $4^{x+2} + 3 = 19$

12) $2^{2+x} - 8 = 24$

13) $3^{x+2} \times 9^{x-2} = 27^{2x+4}$

14) $\dfrac{2^{x+4}}{8^{2x-2}} = 4^{3x+4} \times 32^{x+2}$

Basic Properties

Commutative Property

Addition and Multiplication are commutative: switching the order of two numbers being added or multiplied does not change the result.

Examples:

Associative Property

Addition and multiplication are associative: the order that numbers are grouped in addition and multiplication does not affect the result.

Examples:

Distributive Property

The distributive property of multiplication over addition: multiplication may be distributed over addition.

Examples:

The Zero Property of Addition

Adding 0 to a number leaves it unchanged. We call 0 the additive identity.

Example:

The Zero Property of Multiplication

Multiplying any number by 0 gives 0.

Example:

The Multiplicative Identity

We call 1 the multiplicative identity. Multiplying any number by 1 leaves the number unchanged.

Example:

8. EE. A 2: Evaluate Roots (rational/irrational)
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A perfect square is a number that can be expressed as the *product of two equal integers*.

Examples of perfect squares

- 9
  - 9 is a perfect square because it can be expressed as 3 * 3 (the product of two equal integers)
- 16
  - 16 is a perfect square because it can be expressed as 4 * 4 (the product of two equal integers)
- 25
  - 25 is a perfect square because it can be expressed as 5 * 5 (the product of two equal integers)

Non examples of perfect squares

- 8
  - 8 is a **not** perfect square because you cannot express it as the product of two equal integers
- 5
  - 5 is a **not** perfect square because it cannot be expressed as the product of two equal integers
- 7
  - 7 is a **not** perfect square because you cannot express it as the product of two equal integers

Perfect Square	Factors
1	1*1
4	2*2

A **perfect cube** is the result of multiplying a number three times by itself. $a \cdot a \cdot a = a^3$
We can also say that **perfect cubes** are the numbers that have exact **cube** roots.

Perfect Cube	Factors
1	1*1*1
8	2*2*2

Let's take a look at the some of the pre-test questions:

8. $\sqrt{16}=$	**9.** $\sqrt[3]{(1/27)}$	**10.** **$X^2=25$**
11. **$X^2=4/9$**	**12.** **$X^3=27$**	**13.** **What is the length of a square with an area of 49 in^2?**

Taking this concept further...

$9^{\frac{1}{2}}$ means

In general, here's the way to rewrite a term with a fractional exponent:

$$x^{\frac{a}{b}} = \sqrt[b]{x^a}$$

Evaluate $8^{\frac{2}{3}}$.

Rewrite each term using radical form. Evaluate the root. Evaluate the power.

1. $64^{\frac{1}{2}} =$ ______
= ______
= ______

2. $100^{\frac{1}{2}} =$ ______
= ______
= ______

3. $400^{\frac{1}{2}} =$ ______
= ______
= ______

4. $64^{\frac{2}{3}} =$ ______
= ______
= ______

5. $216^{\frac{2}{3}} =$ ______
= ______
= ______

6. $1000^{\frac{2}{3}} =$ ______
= ______
= ______

7. $625^{\frac{3}{4}} =$ ______
= ______
= ______

8. $32^{\frac{2}{5}} =$ ______
= ______
= ______

9. $10{,}000^{\frac{5}{4}} =$ ______
= ______
= ______

8. EE. A. 3 and 4: Scientific Notation
Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.

Scientific Notation

Scientific Notation Keeping track of place value in very large or very small numbers written in standard form may be difficult. It is more efficient to write such numbers in scientific notation. A number is expressed in scientific notation when it is written as a product of two factors, one factor that is greater than or equal to 1 and less than 10 and one factor that is a power of ten.

Scientific Notation	A number is in scientific notation when it is in the form $a \times 10^n$, where $1 \leq a < 10$ and n is an integer.

Let's take a look at the some of the pre-test questions:

14. Write 0.0000429 in scientific notation.	15. Express 2.45 x 10^5 in standard form.	16. Multiply. Leave your final answer in scientific notation. $(6.45*10^{11})(3.2*10^4)$

17. Divide. Leave your final answer in scientific notation. No negative exponents in the denominator. $(3.45*10^5)/(6.7*10^{-2})$	18. The speed of light is $3.5*10^8$ meters per second. If the sun is $1.5*10^{11}$ meters from earth, how many seconds does it take the light to reach the earth? Express your answer in scientific notation.	**BLANK SPACE**

Example 1 Express 3.52×10^4 in standard notation.

Example 2 Express 6.21×10^{-5} in standard notation.

Example 3 Express 37,600,000 in scientific notation.

Example 4 Express 0.0000549 in scientific notation.

Products and Quotients with Scientific Notation You can use properties of powers to compute with numbers written in scientific notation.

Example 1 Evaluate $(6.7 \times 10^3)(2 \times 10^{-5})$. Express the result in scientific and standard notation.

Example 2 Evaluate $\frac{1.5088 \times 10^8}{4.1 \times 10^5}$. Express the result in scientific and standard notation.

Feel free to use the next page to show work.

Scientific Notation

***Part A:* Express each of the following in standard form.**

1. 9.65×10^{-4}

2. 6.452×10^{2}

3. 8.5×10^{-2}

4. 8.77×10^{-1}

5. 2.71×10^{4}

6. 6.4×10^{-3}

***Part B:* Express each of the following in scientific notation.**

7. 78,000

8. 16

9. 0.00053×10^{6}

10. 0.0043

11. 250

12. 0.875×10^{-3}

***Part C:* Use the associative property to simplify. Express your final answer in scientific notation rounded to the nearest tenth.**

13. $(6.02 \times 10^{23})(8.65 \times 10^{4})$

14. $\dfrac{(5.4 \times 10^{4})(2.2 \times 10^{7})}{4.5 \times 10^{5}}$

15. $(6.02 \times 10^{23})(9.63 \times 10^{-2})$

16. $\dfrac{(6.02 \times 10^{23})(-1.42 \times 10^{-15})}{6.54 \times 10^{-6}}$

17. $\dfrac{5.6 \times 10^{-18}}{8.9 \times 10^{8}}$

18. $\dfrac{(6.02 \times 10^{23})(-5.11 \times 10^{-27})}{-8.23 \times 10^{5}}$

19. $(-4.12 \times 10^{-4})(7.33 \times 10^{12})$

20. $\dfrac{(3.1 \times 10^{14})(4.4 \times 10^{-12})}{-6.6 \times 10^{-14}}$

21. A cubic millimeter of blood contains about 5×10^{6} red blood cells. An adult human body contains approximately 5×10^{6} cubic millimeters of blood. About how many red blood cells does a human body contain?

22. A liter of healthy human blood contains approximately 4×10^{9} white blood cells. A healthy adult contains about 5.5 liters of blood. How many more times red blood cells does a healthy adult have than white blood cells. *Refer to previous answer for the number of red blood cells.* Express your final answer rounded to the nearest tenth in scientific notation.

You may use this page to show work for the previous page: